

Cryptography

ECE5632 - Spring 2024

Lecture 7B

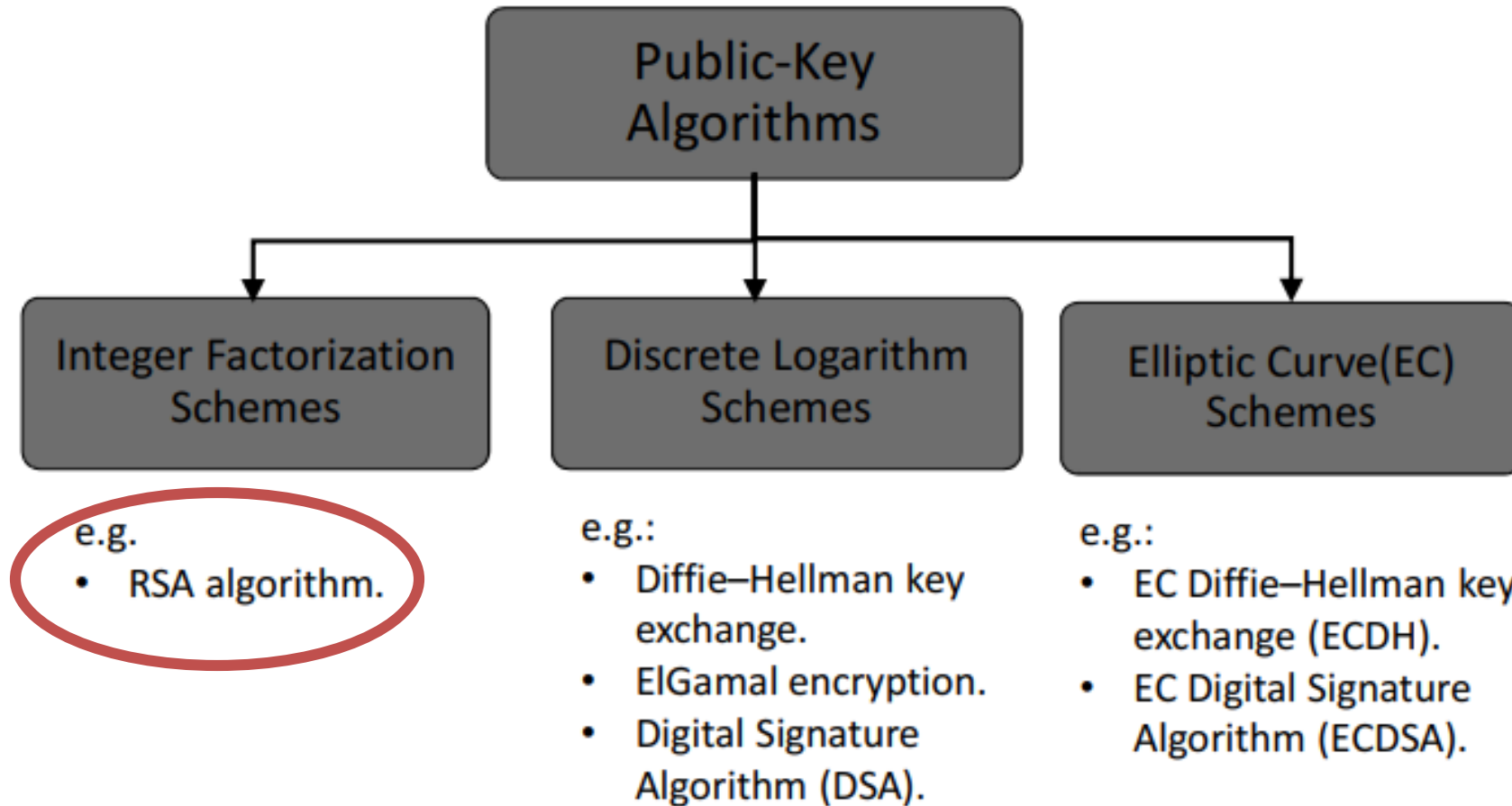
Dr. Farah Raad



Lecture Topic

RSA Algorithm & Diffie-Hellman Key

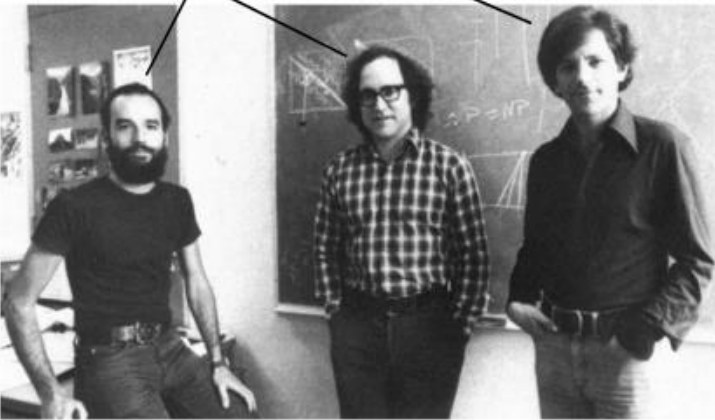
PKC Algorithms: Three Families



RSA Algorithm

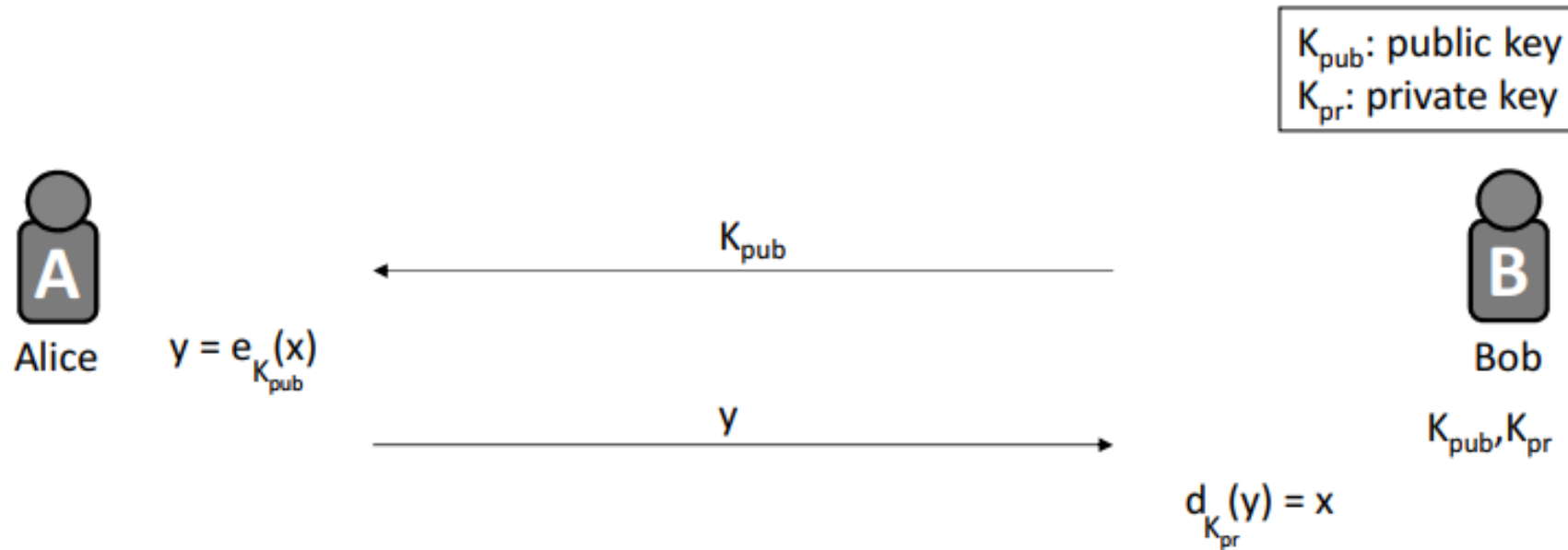
➤ Some History

- ✓ 1976: Public key cryptography first introduced:
Martin Hellman and Whitfield Diffie published their landmark publickey paper in 1976
- ✓ 1977: **R**ivest–**S**hamir–**A**dleman (RSA) proposed the asymmetric RSA cryptosystem algorithm



- ✓ Until now, RSA is the most widely use asymmetric cryptosystem although elliptic curve cryptography (ECC) becomes increasingly popular
- ✓ RSA is mainly used for two applications
 - Transport of (i.e., symmetric) keys
 - Digital signatures

RSA Algorithm



- 1) How to encrypt/decrypt?
- 2) How to compute K_{pub} and K_{pr} ?

RSA Algorithm

➤ Encryption and Decryption

- ✓ RSA operations are done over the integer ring Z_n (i.e., arithmetic modulo n), where $n = p * q$, with p, q being large primes
- ✓ Encryption and decryption are simply exponentiations in the ring
- ✓ In practice x, y, n and d are very long integer numbers (≥ 1024 bits)

Definition

Given the public key $(n, e) = k_{pub}$ and the private key $d = k_{pr}$ we write

$$y = e_{k_{pub}}(x) \equiv x^e \pmod{n}$$

$$x = d_{k_{pr}}(y) \equiv y^d \pmod{n}$$

where $x, y \in Z_n$.

We call $e_{k_{pub}}()$ the encryption and $d_{k_{pr}}()$ the decryption operation.

RSA Algorithm

➤ Key Generation

- ✓ Like all asymmetric schemes, RSA has set-up phase during which the private and public keys are computed

Algorithm: RSA Key Generation

Output: public key: $k_{pub} = (n, e)$ and private key $k_{pr} = d$

1. Choose two large primes p, q
2. Compute $n = p * q$
3. Compute $\Phi(n) = (p-1) * (q-1)$
4. Select the public exponent $e \in \{1, 2, \dots, \Phi(n)-1\}$ such that $\gcd(e, \Phi(n)) = 1$
5. Compute the private key d such that $d * e \equiv 1 \text{ mod } \Phi(n)$
6. **RETURN** $k_{pub} = (n, e), k_{pr} = d$

RSA Algorithm

Remarks:

- Choosing two large, distinct primes p, q (in Step 1) is non-trivial
- $\gcd(e, \Phi(n)) = 1$ ensures that e has an inverse and, thus, that there is always a private key d

Notes:

- ✓ In practice, n is ≥ 1024 bits long.
- ✓ Strength of RSA with $n = 2^{3072}$ is equivalent to AES128.
- ✓ Longer n means more security, but slower computation.
- ✓ p and q should differ in length by only a few digits . . . $p, q \geq 512$ bits long



RSA Algorithm

Example: RSA with small numbers:



Alice

Message $x = 4$

$$\begin{aligned} y &= e_{K_{\text{pub}}}(x) \equiv x^e \pmod{n} \\ &\equiv 4^3 \pmod{33} \\ &\equiv 31 \pmod{33} \end{aligned}$$

$K_{\text{pub}} = (33, 3)$

y



Bob

1. $p=3, q=11$
2. $n = 33$
3. $\Phi(n) = (p-1) \cdot (q-1)$
 $= 2 \cdot 10 = 20$
4. Choose $e = 3$
5. $d \equiv e^{-1} \equiv 7 \pmod{20}$

$$\begin{aligned} x &= d_{K_{\text{pr}}}(y) \equiv y^d \pmod{n} \\ &\equiv 31^7 \pmod{33} \\ &\equiv 4 \pmod{33} \end{aligned}$$

RSA Algorithm

Parameters Example

Example of practical RSA
parameters for $n = 1024 \rightarrow$

$p =$ E0DFD2C2A288ACEBC705EFAB30E4447541A8C5A47A37185C5A9
CB98389CE4DE19199AA3069B404FD98C801568CB9170EB712BF
10B4955CE9C9DC8CE6855C6123_h
 $q =$ EBE0FCF21866FD9A9F0D72F7994875A8D92E67AEE4B515136B2
A778A8048B149828AEA30BD0BA34B977982A3D42168F594CA99
F3981DDABFAB2369F229640115_h
 $n =$ CF33188211FDF6052BDBB1A37235E0ABB5978A45C71FD381A91
AD12FC76DA0544C47568AC83D855D47CA8D8A779579AB72E635
D0B0AAAC22D28341E998E90F82122A2C06090F43A37E0203C2B
72E401FD06890EC8EAD4F07E686E906F01B2468AE7B30CBD670
255C1FEDE1A2762CF4392C0759499CC0ABECFF008728D9A11ADF_h
 $e =$ 40B028E1E4CCF07537643101FF72444A0BE1D7682F1EDB553E3
AB4F6DD8293CA1945DB12D796AE9244D60565C2EB692A89B888
1D58D278562ED60066DD8211E67315CF89857167206120405B0
8B54D10D4EC4ED4253C75FA74098FE3F7FB751FF5121353C554
391E114C85B56A9725E9BD5685D6C9C7EED8EE442366353DC39_h
 $d =$ C21A93EE751A8D4FBFD77285D79D6768C58EBF283743D2889A3
95F266C78F4A28E86F545960C2CE01EB8AD5246905163B28D0B
8BAABB959CC03F4EC499186168AE9ED6D88058898907E61C7CC
CC584D65D801CFE32DFC983707F87F5AA6AE4B9E77B9CE630E2
C0DF05841B5E4984D059A35D7270D500514891F7B77B804BED81_h



RSA Algorithm

Proof of Correctness

We need to prove that $d_{K_{pr}}(e_{K_{pub}}(x)) = x$

i.e., prove that $(x^e)^d \equiv x^{de} \pmod{n} \equiv x \pmod{n}$



RSA Algorithm: Practical Consideration

RSA is a heavy user of exponentiation...

Encryption: $y = x^e \bmod n$

Decryption: $x = y^d \bmod n$

Problem: How to quickly exponentiate with extremely large numbers?

Solution: Using Square-and-Multiply Algorithm

	e.g., x^{26}
Square	SQ $x \cdot x = x^2$
Multiply	MUL $x \cdot x^2 = x^3$
	SQ $x^3 \cdot x^3 = x^6$
	SQ $x^6 \cdot x^6 = x^{12}$
	MUL $x \cdot x^{12} = x^{13}$
	SQ $x^{13} \cdot x^{13} = x^{26}$

Represent the exponent in binary; x^{11010}

Initially start with x^1

$$(x^1)^2 = x^2$$

$$(x^2)x = x^3$$

$$(x^3)^2 = x^6$$

$$(x^6)^2 = x^{12}$$

$$(x^{12})x = x^{13}$$

$$(x^{13})^2 = x^{26}$$

Simply construct the binary exponent from left to right by shifting (SQ) and adding (MUL).

Note: MUL is only used for the 1 bits.

Looks random.. How do I know when to SQ and when to MUL?



Security of RSA

The RSA discussed so far is called schoolbook RSA.

It has several weaknesses:

- RSA is deterministic.
- $y=x$ for $x=0, 1, -1$.
- RSA is malleable.

A malleable cipher allows an attacker to modify the value of x without decrypting y .
e.g., attacker wants to multiply x by $s=2$. But only has access to y and e .

Then replacing y with $s^e y$ leads to..

$$\text{decryption: } (s^e y)^d \equiv s^{ed} x^{ed} \equiv sx \pmod{n}.$$

In practice, these weaknesses can be eliminated by using padding.
i.e., Optimal Asymmetric Encryption Padding (OAEP)



Security of RSA: Attacks

Attack possibilities against RSA:

1. Protocol attacks
2. Mathematical attacks. i.e., factoring the modulus.

The attackers knows n and e .

But can't compute d because p and q are unknown!

Longer n is more difficult to factor, but slower algorithm.

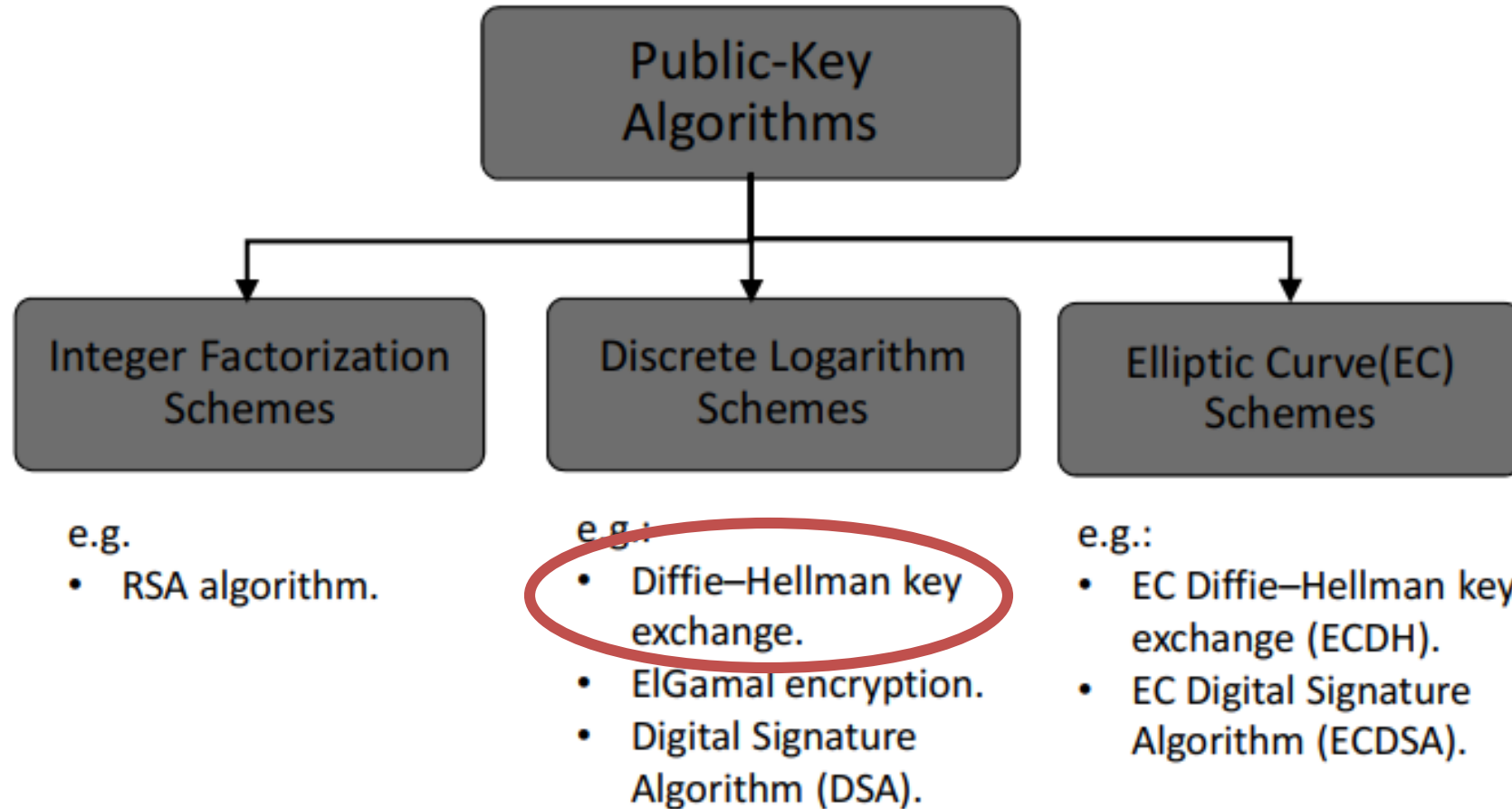
Minimum n size = 1024. Recommended 2048-4096.

3. Side-channel attacks.

Exploit info leaked from the processing power or time (i.e., physical channels)



PKC Algorithms: Three Families



Diffie-Hellman Key Exchange (DHKE)

- Proposed in 1976 by Whitfield Diffie and Martin Hellman
- Widely used, e.g. in Secure Shell (SSH), Transport Layer Security (TLS), and Internet Protocol Security (IPSec)
- The Diffie–Hellman Key Exchange (DHKE) is a key exchange protocol and not used for encryption
- (For the purpose of encryption based on the DHKE, ElGamal can be used.)

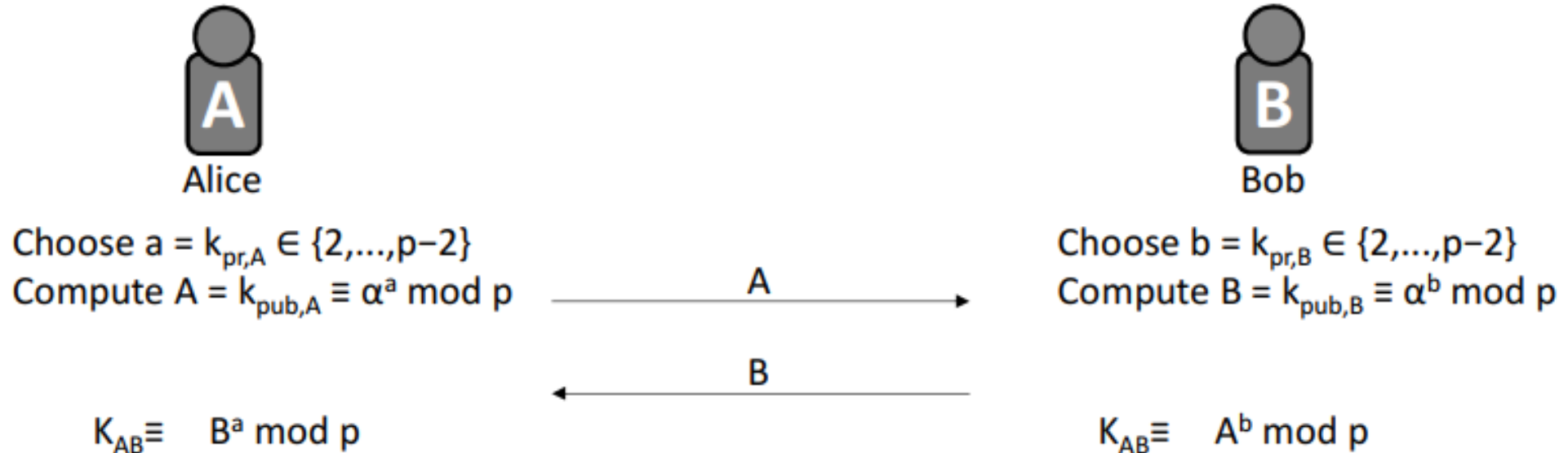


Diffie-Hellman Key Exchange (DHKE)

Diffie-Hellman setup:

1. Choose a large prime p .
2. Choose an integer $\alpha \in \{2, 3, \dots, p-2\}$.
3. Publish p and α .

p is a large prime ≥ 1024 bits long.
We'll soon discuss the nature of α .



As a result, K_{AB} is the shared secret.
e.g., we can use the 128 MSB of K_{AB} as a key for AES128.

Diffie-Hellman Key Exchange (DHKE)

Essential idea:

- Choose two random secrets **a** and **b**

$$(\alpha^a)^b \bmod p = (\alpha^b)^a \bmod p$$

- Both parties can calculate that value without sending secrets over the wire



Diffie-Hellman Key Exchange (DHKE)

Alice

Choose random private key
 $k_{prA} = a \in \{1, 2, \dots, p-1\}$

Compute corresponding public key
 $k_{pubA} = A = \alpha^a \bmod p$

Compute common secret
 $k_{AB} = B^a = (\alpha^b)^a \bmod p$

Bob

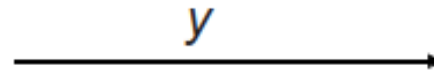
Choose random private key
 $k_{prB} = b \in \{1, 2, \dots, p-1\}$

Compute corresponding public key
 $k_{pubB} = B = \alpha^b \bmod p$

Compute common secret
 $k_{AB} = A^b = (\alpha^a)^b \bmod p$

We can now use the joint key k_{AB}
for encryption, e.g., with AES

$$y = AES_{k_{AB}}(x)$$

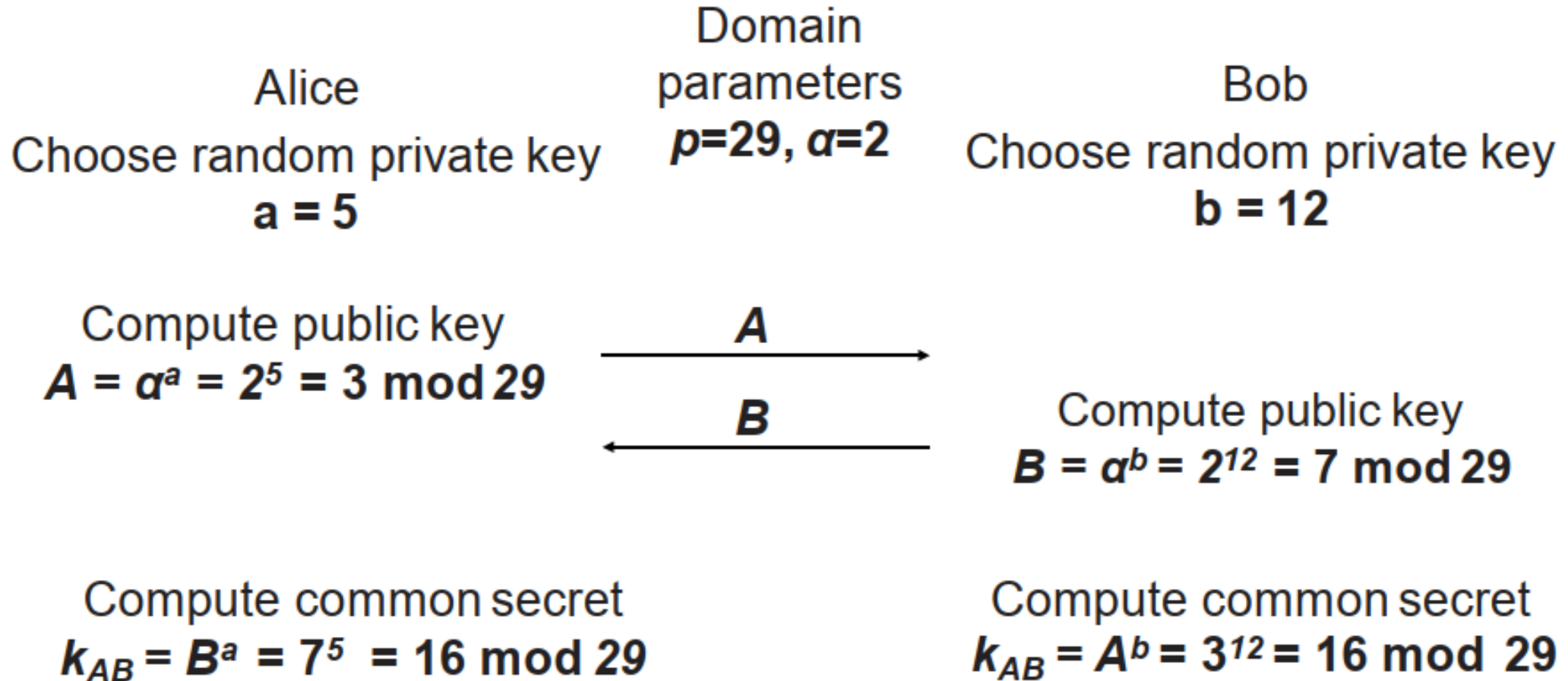


$$x = AES_{k_{AB}}^{-1}(y)$$



Diffie-Hellman Key Exchange (DHKE)

Example



Diffie-Hellman Key Exchange (DHKE)

So, . . . $K_{AB} \equiv B^a \bmod p \equiv A^b \bmod p$

$$A \equiv \alpha^a \bmod p$$

$$B \equiv \alpha^b \bmod p$$

How is that possible??



Proof:

$$B^a \equiv (\alpha^b)^a \equiv \alpha^{ab} \bmod p$$

$$A^b \equiv (\alpha^a)^b \equiv \alpha^{ab} \bmod p$$

Very simple. Very important.

α must be a primitive element.

What that means? Time for some math...



Groups

Cyclic Groups



Revisiting Groups

Group $(G, \circ)$: a set of elements, with 1 group operator.

E.g., :

$(G, +)$ additive group

$(G, \times)$ multiplicative group

Has certain properties that must be satisfied:

A1. Closure:

If a and b belong to G , then $a \circ b$ is also in G .

A2. Associativity:

$a \circ (b \circ c) = (a \circ b) \circ c$ for all a, b, c in G

M1...

etc. . .

❑ See Lecture 6A.

Revisiting Groups

Theorem 8.2.1

The set $\mathbb{Z}_n^$ which consists of all integers $i = 0, 1, \dots, n-1$ for which $\gcd(i, n) = 1$ forms an abelian group under multiplication modulo n . The identity element is $e = 1$.*

Example Let us verify the validity of the theorem by considering the following example:

If we choose $n = 9$, $\mathbb{Z}_9^*$ consists of the elements $\{1, 2, 4, 5, 7, 8\}$.

Multiplication table for $\mathbb{Z}_9^*$

$\times \bmod 9$	1	2	4	5	7	8
1	1	2	4	5	7	8
2	2	4	8	1	5	7
4	4	8	7	2	1	5
5	5	1	2	7	8	4
7	7	5	1	8	4	2
8	8	7	5	4	2	1

Revisiting Groups

Example : Is $(\mathbb{Z}_9, \times)$ a multiplicative group?

$$\mathbb{Z}_9 = \{\textcircled{0}, 1, 2, \textcircled{3}, 4, 5, \textcircled{6}, 7, 8\}$$

Check for property A1, A2, M1, etc..

·
·

Problem with inverse property: Inverses only exist for elements a ; $\gcd(a, 9) = 1$

$\therefore$ elements 0, 3, 6 have no inverse in $\mathbb{Z}_9$.

So, we'll define a special set called $\mathbb{Z}_n^*$, by simply removing noninvertible elements.
The elements of $\mathbb{Z}_n^*$ still satisfy all properties of a group.

i.e., $\mathbb{Z}_9^* = \{1, 2, 4, 5, 7, 8\}$ is a multiplicative group.

$|G|$ = Order of G : The number of elements in G a.k.a. the cardinality of G .

$$\therefore |\mathbb{Z}_9^*| = 6$$



Cyclic Groups

Definition 8.2.2 Finite Group

A group $(G, \circ)$ is finite if it has a finite number of elements. We denote the cardinality or order of the group G by $|G|$.

- $(\mathbb{Z}_n, +)$: the cardinality of $\mathbb{Z}_n$ is $|\mathbb{Z}_n| = n$ since $\mathbb{Z}_n = \{0, 1, 2, \dots, n-1\}$.
- $(\mathbb{Z}_n^*, \cdot)$: remember that $\mathbb{Z}_n^*$ is defined as the set of positive integers smaller than n which are relatively prime to n . Thus, the cardinality of $\mathbb{Z}_n^*$ equals Euler's phi function evaluated for n , i.e., $|\mathbb{Z}_n^*| = \Phi(n)$. For instance, the group $\mathbb{Z}_9^*$ has a cardinality of $\Phi(9) = 3^2 - 3^1 = 6$. This can be verified by the earlier example where we saw that the group consist of the six elements $\{1, 2, 4, 5, 7, 8\}$.

Cyclic Groups

Definition 8.2.3 Order of an element

The order $\text{ord}(a)$ of an element a of a group $(G, \circ)$ is the smallest positive integer k such that

$$a^k = \underbrace{a \circ a \circ \dots \circ a}_{k \text{ times}} = 1,$$

where 1 is the identity element of G .

- In the previous example, $\text{ord}(3)=5$.
- Don't confuse $\text{ord}(a)$ with $|G|$

Cyclic Groups

Example We try to determine the order of $a = 3$ in the group $\mathbb{Z}_{11}^*$. For this, we keep computing powers of a until we obtain the identity element 1.

$$a^1 = 3$$

$$a^2 = a \cdot a = 3 \cdot 3 = 9$$

$$a^3 = a^2 \cdot a = 9 \cdot 3 = 27 \equiv 5 \pmod{11}$$

$$a^4 = a^3 \cdot a = 5 \cdot 3 = 15 \equiv 4 \pmod{11}$$

$$a^5 = a^4 \cdot a = 4 \cdot 3 = 12 \equiv 1 \pmod{11}$$

From the last line it follows that $\text{ord}(3) = 5$.

$$a^6 = a^5 \cdot a \equiv 1 \cdot a \equiv 3 \pmod{11}$$

$$a^7 = a^5 \cdot a^2 \equiv 1 \cdot a^2 \equiv 9 \pmod{11}$$

$$a^8 = a^5 \cdot a^3 \equiv 1 \cdot a^3 \equiv 5 \pmod{11}$$

$$a^9 = a^5 \cdot a^4 \equiv 1 \cdot a^4 \equiv 4 \pmod{11}$$

$$a^{10} = a^5 \cdot a^5 \equiv 1 \cdot 1 \equiv 1 \pmod{11}$$

$$a^{11} = a^{10} \cdot a \equiv 1 \cdot a \equiv 3 \pmod{11}$$

$\vdots$

the powers of a run through the sequence $\{3, 9, 5, 4, 1\}$

Cyclic Groups

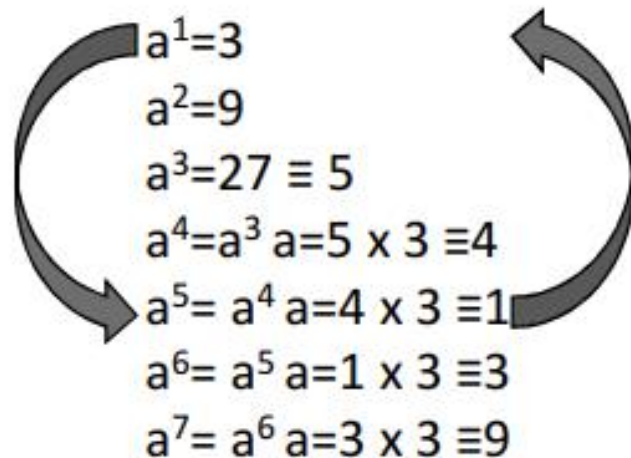


In case of the multiplicative group Z_p^* , where p is prime;

$$\therefore Z_p^* = \{1, 2, 3, \dots, p-1\}$$

$$\text{e.g., } Z_{11}^* = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

To understand what's cyclic groups,
let's pick a number ($a=3$) and compute all its powers..



The result cycles over and over again.

Cyclic Groups

Definition 8.2.4 Cyclic Group

A group G which contains an element α with maximum order $\text{ord}(\alpha) = |G|$ is said to be cyclic. Elements with maximum order are called primitive elements or generators.



Cyclic Groups

Example We want to check whether $a = 2$ happens to be a primitive element of $\mathbb{Z}_{11}^* = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

$$a = 2$$

$$a^2 = 4$$

$$a^3 = 8$$

$$a^4 \equiv 5 \pmod{11}$$

$$a^5 \equiv 10 \pmod{11}$$

$$a^6 \equiv 9 \pmod{11}$$

$$a^7 \equiv 7 \pmod{11}$$

$$a^8 \equiv 3 \pmod{11}$$

$$a^9 \equiv 6 \pmod{11}$$

$$a^{10} \equiv 1 \pmod{11}$$

$$\text{ord}(a) = 10 = |\mathbb{Z}_{11}^*|.$$

Note that the cardinality of the group is $|\mathbb{Z}_{11}^*| = 10$.

❖ Let's look again at all the elements that are generated by powers of two.

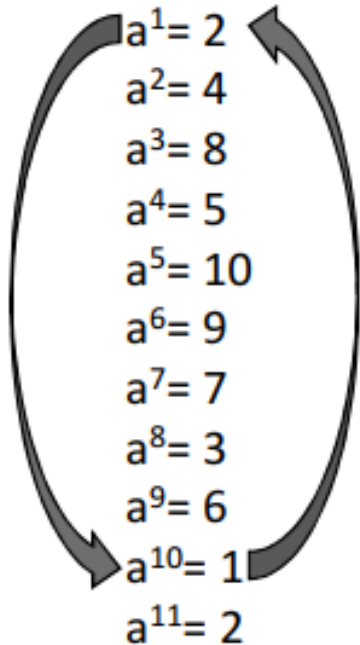
i	1	2	3	4	5	6	7	8	9	10
a^i	2	4	8	5	10	9	7	3	6	1

✓ The powers of $a = 2$ actually generate all elements of the group $\mathbb{Z}_{11}^*$

Cyclic Groups



$$Z_{11}^* = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$



Which elements did the number 2 **generate**? . . . All of them.
So, we call it a *generator*, primitive root, or a *primitive element*.

$$\therefore \text{ord}(2)=10$$

$a=2$ is a generator of Z_{11}^*

$$\therefore 2^{10} \bmod 11 \equiv 1$$

$$\therefore 2^{45363457210} \bmod 11 \equiv 1.$$

❖ It is important to stress that the number 2 is not necessarily a generator in other cyclic groups

$$\mathbb{Z}_7^*, \text{ord}(2) = 3$$

✓ The element 2 is thus not a generator in that group.

Cyclic Groups

- Cyclic Groups are the basis of several cryptosystems.
 - For every prime p , $(\mathbb{Z}_p^*, \times)$ is a cyclic group.

Theorem 8.2.2 *For every prime p , $(\mathbb{Z}_p^*, \cdot)$ is an abelian finite cyclic group.*

Theorem 8.2.3

Let G be a finite group. Then for every $a \in G$ it holds that:

1. $a^{|G|} = 1$
2. $\text{ord}(a)$ divides $|G|$

Cyclic Groups

Theorem 8.2.3

Let G be a finite group. Then for every $a \in G$ it holds that:

1. $a^{|G|} = 1$
2. $\text{ord}(a)$ divides $|G|$

➤ **Property 1:** Proof using Fermat's little theorem for $\mathbb{Z}_p^*$

$$a^p \equiv a \pmod{p}$$

$$a^{p-1} \equiv 1 \pmod{p}$$

$$|\mathbb{Z}_p^*| = p-1$$

$$a^{p-1} = a^{|\mathbb{Z}_p^*|} = 1$$

➤ **Property 2:** example using $\mathbb{Z}_{11}^*$

$$|\mathbb{Z}_{11}^*| = 10$$

Possible orders $\in \{1, 2, 5, 10\}$

Cyclic Groups



Theorem 8.2.3

Let G be a finite group. Then for every $a \in G$ it holds that:

1. $a^{|G|} = 1$
2. $\text{ord}(a)$ divides $|G|$

➤ **Property 2:** example using $\mathbb{Z}_{11}^*$

$$|\mathbb{Z}_{11}^*| = 10$$

Possible orders $\in \{1, 2, 5, 10\}$

➤ How many primitive elements (i.e., generators) do we have?

✓ **Four elements: 2, 6, 7, 8.**

✓ The only element orders in this group are 1, 2, 5, and 10, since these are the only integers that divide 10.

$\text{ord}(1)$	$=$	1
$\text{ord}(2)$	$=$	10
$\text{ord}(3)$	$=$	5
$\text{ord}(4)$	$=$	5
$\text{ord}(5)$	$=$	5
$\text{ord}(6)$	$=$	10
$\text{ord}(7)$	$=$	10
$\text{ord}(8)$	$=$	10
$\text{ord}(9)$	$=$	5
$\text{ord}(10)$	$=$	2



Cyclic Groups

➤ How is this related to DHKE?

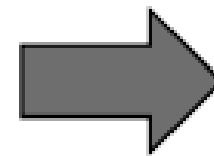
- ✓ Cyclic groups make good Discrete Logarithm Problems.

Definition: Discrete Logarithm Problem (DLP)

Given a prime p , an element $\beta \in \mathbb{Z}_p^*$, and the generator α ,

find x such that; $\alpha^x \equiv \beta \pmod{p}$

e.g., In DHKE, attackers know p , α , A , B
However, finding $K_{AB} = \alpha^{ab}$ is a hard problem.



Diffie-Hellman Problem (DHP)

Especially with a large p , attackers need to compute $\log_{\alpha} B \pmod{p}$.



Discrete Logarithm Problem (DLP)

Definition 8.3.1 Discrete Logarithm Problem (DLP) in $\mathbb{Z}_p^*$
Given is the finite cyclic group $\mathbb{Z}_p^$ of order $p - 1$ and a primitive element $\alpha \in \mathbb{Z}_p^*$ and another element $\beta \in \mathbb{Z}_p^*$. The DLP is the problem of determining the integer $1 \leq x \leq p - 1$ such that:*

$$\alpha^x \equiv \beta \pmod{p}$$

$$x = \log_{\alpha} \beta \pmod{p}.$$



Discrete Logarithm Problem (DLP)

In other words...

If x is known, it's computationally easy to get $\alpha^x \equiv \beta \pmod{p}$

However, for large parameters, it's very difficult to get $\log_{\alpha} \beta \pmod{p}$

This forms a one-way function.

e. g., Z_{47}^* , $\beta = 41$, $\alpha = 5$

Find x such that $5^x \equiv 41 \pmod{47}$.

Using brute force, $x = 15$.

$$2^x \equiv 36 \pmod{47}$$

By using a brute-force attack, we obtain a solution for $x = 17$

Example: mod 7

- 3 is a **primitive element** or **generator** under the **multiplication** operation

$$3^1 = 3 \pmod{7}$$

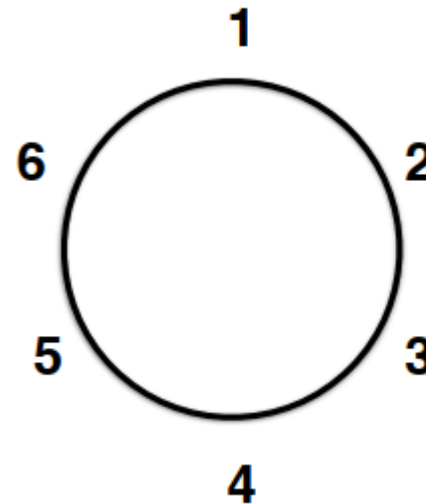
$$3^2 = 9 = 2 \pmod{7}$$

$$3^3 = 27 = 6 \pmod{7}$$

$$3^4 = 81 = 4 \pmod{7}$$

$$3^5 = 243 = 5 \pmod{7}$$

$$3^6 = 729 = 1 \pmod{7}$$



Example: mod 7

```
>>> for i in range(1,7):  
[...     print 3, "**", i, "= ", (3**i) % 7, "mod 7"  
[...  
3 ** 1 = 3 mod 7  
3 ** 2 = 2 mod 7  
3 ** 3 = 6 mod 7  
3 ** 4 = 4 mod 7  
3 ** 5 = 5 mod 7  
3 ** 6 = 1 mod 7
```

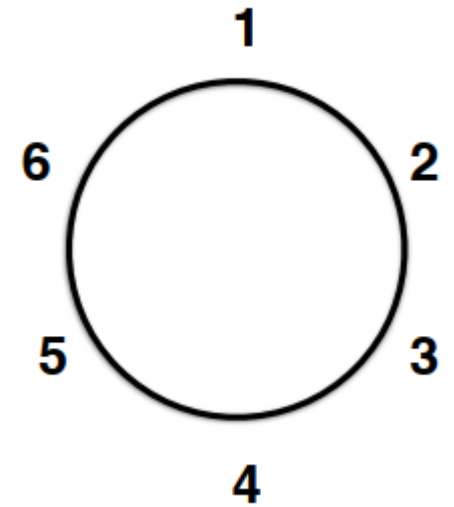
$$\alpha = 3$$

$$\text{DLP: } 3^x = 4 \text{ mod } 7$$

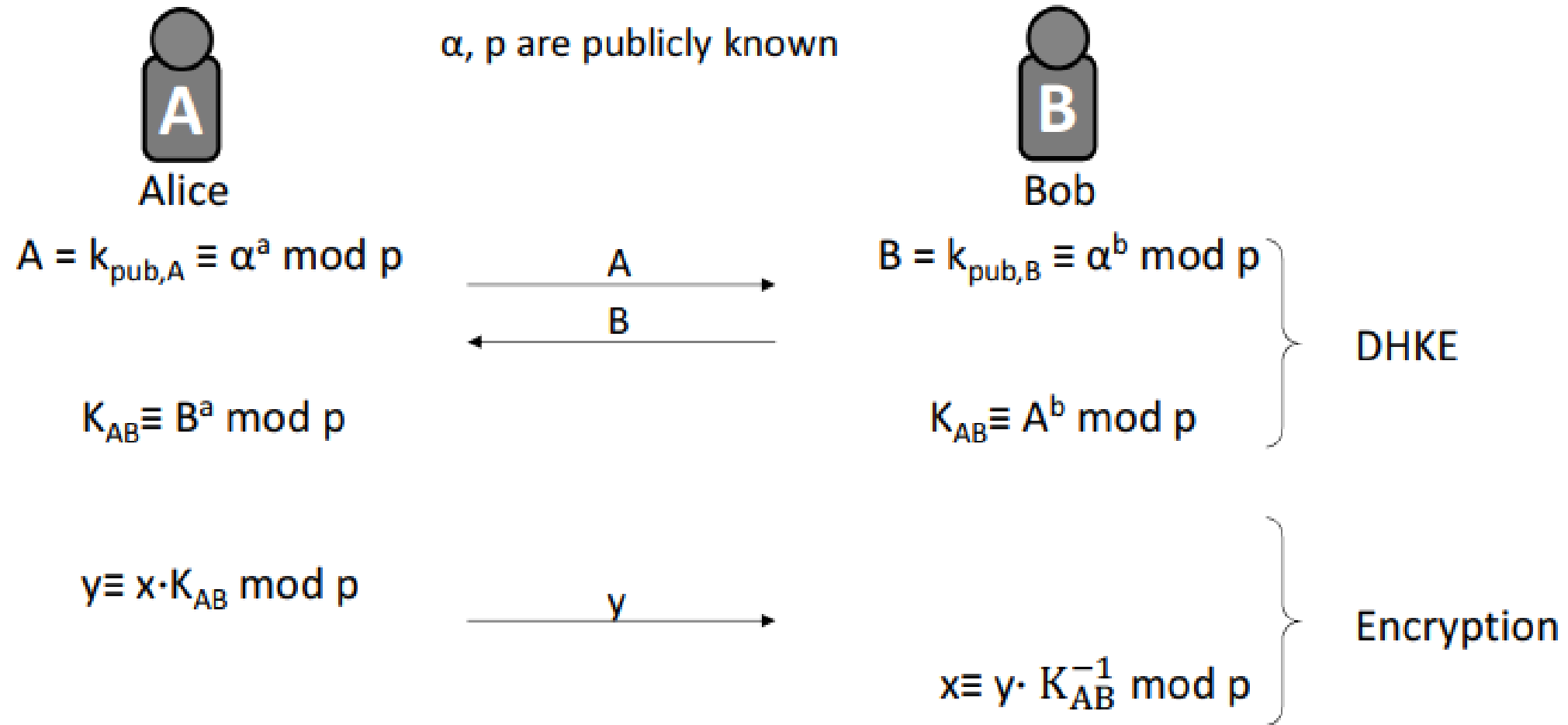
$$x = 4$$

$$\text{DLP: } 3^x = 1 \text{ mod } 7$$

$$x = 6$$



Concept of Encryption using DLP



Diffie-Hellman Problem (DHP)

Attackers know p, α, A, B
Attackers want $K_{AB} = \alpha^{ab}$

- Attacker's possible steps to solve DHP:
 1. Compute $a = \log_{\alpha} A \bmod p$
 2. Compute $B^a = K_{AB} \bmod p$
- For attackers, step 1 is computationally a very hard problem if p is large enough >1024 bits.

Security of DHKE

➤ DHKE alone is vulnerable to active attacks.

- i.e., the protocol can be defeated if the attacker can modify the messages or generate false messages.
- So, digital signatures and public-key certificates are used to overcome this vulnerability.

➤ Passive attacks.

❑ Examples:

- Exhaustive search
- Index-calculus algorithm
- Baby-step giant-step algorithm
- Pollard's rho algorithm
- Pohlig–Hellman algorithm

❑ To overcome, use large p





Thank You!

See You next Lectures!!
Any Question?

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