MSA UNIVERSITY
جامعة أكتوبر للعلوم الحديثة والآداب

The First British Higher Education in Egypt

Cryptography
ECE5632 - Spring 2026

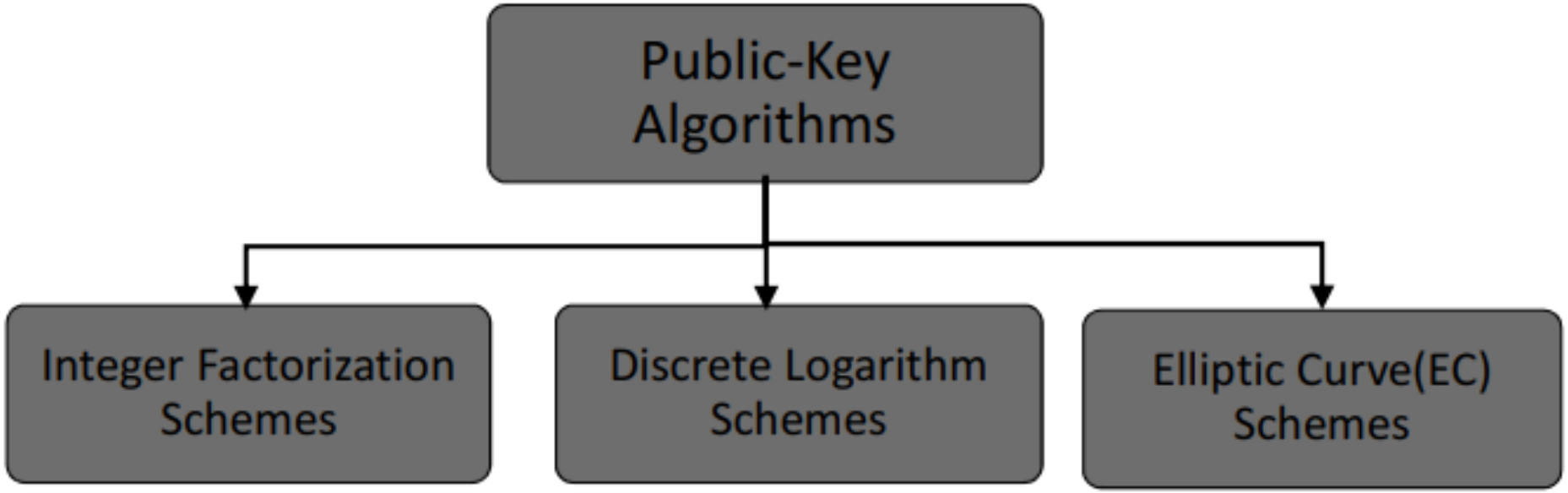
Lecture 8A

Dr. Farah Raad

Lecture Topic

ElGamal Encryption

PKC Algorithms: Three Families



e.g.
- RSA algorithm.

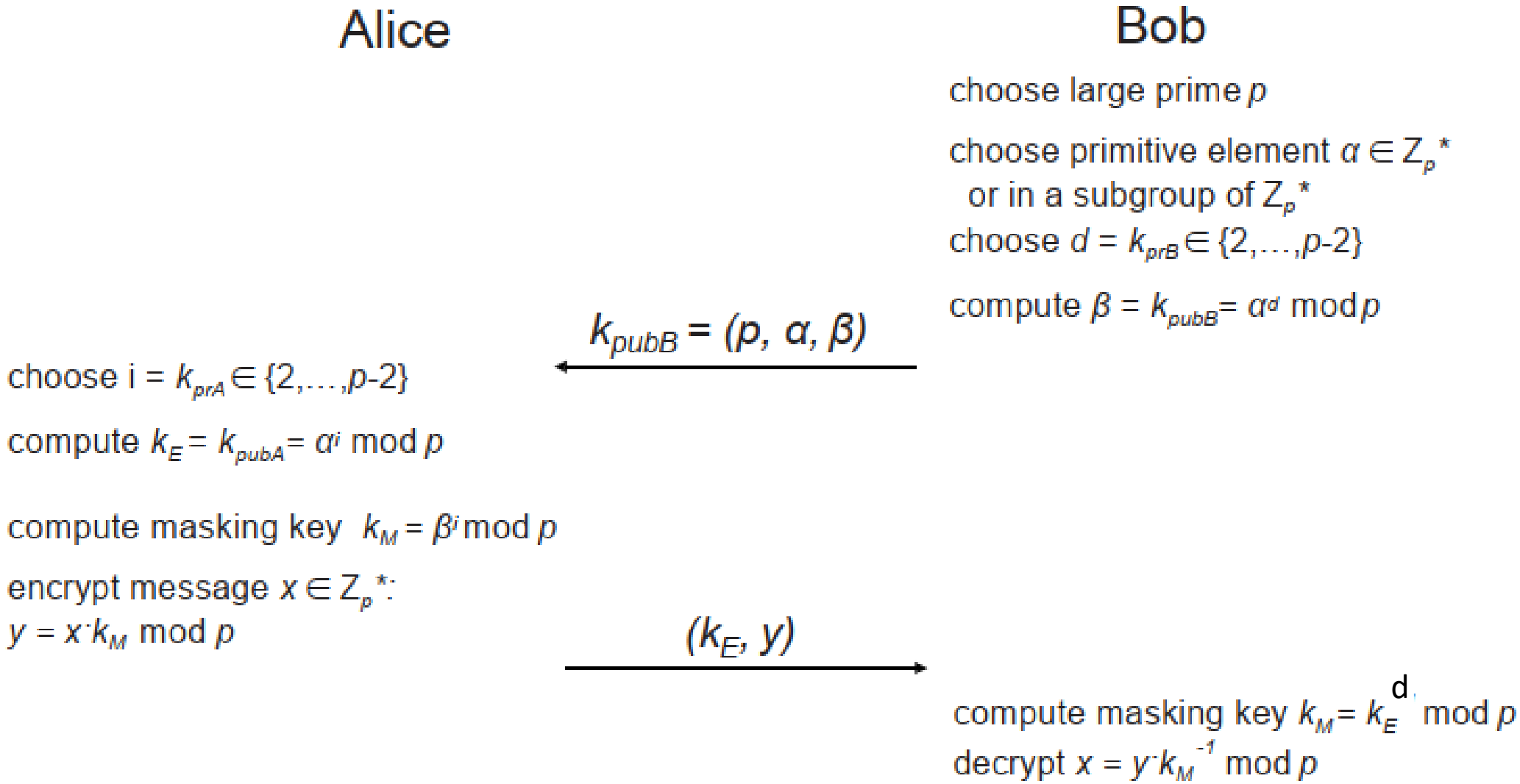
e.g.:
- Diffie–Hellman key exchange.
- ElGamal encryption.
- Digital Signature Algorithm (DSA).

e.g.:
- EC Diffie–Hellman key exchange (ECDH).
- EC Digital Signature Algorithm (ECDSA).

ElGamal Encryption

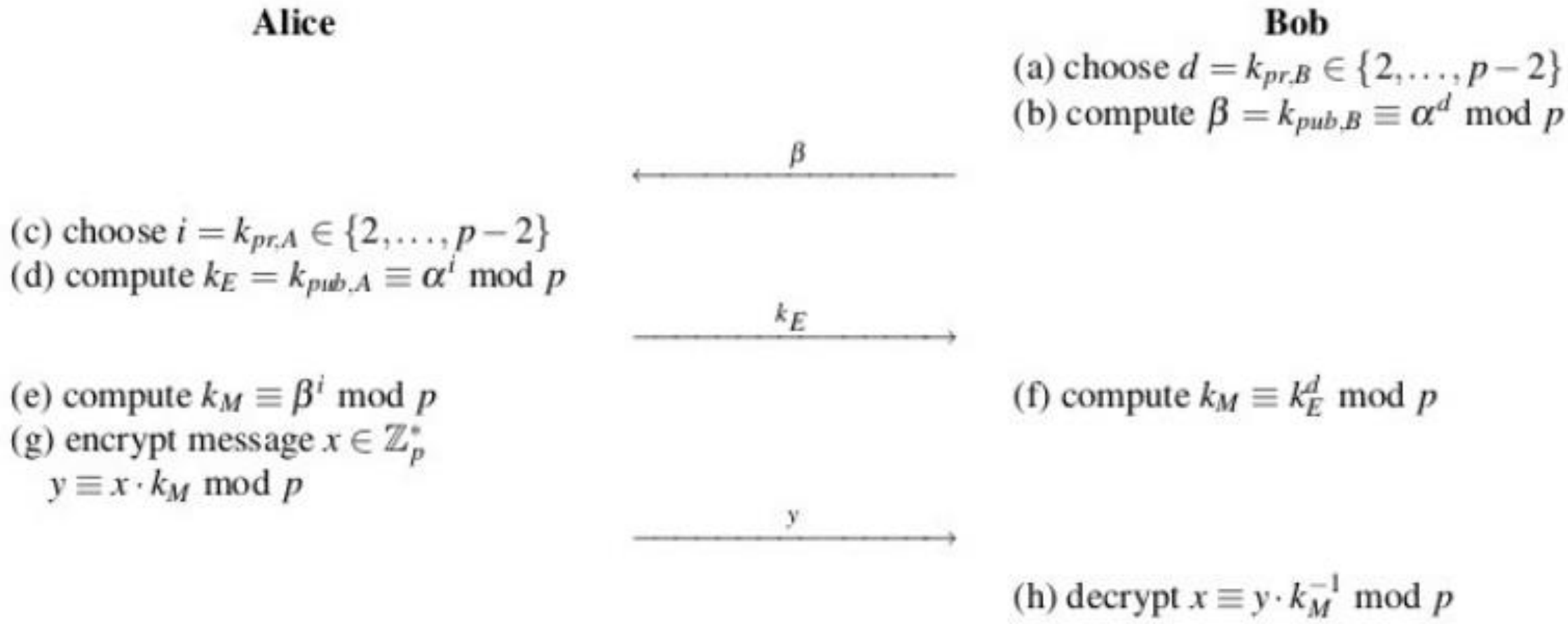
- Invented in 1985 by Taher ElGamal.
- Can be viewed as an extension of the DHKE protocol
- Based on the intractability of the discrete logarithm problem and the Diffie–Hellman problem

The Elgamal Encryption Protocol



✓ This looks very similar to the DHKE! The actual Elgamal protocol re-orders the computations which helps to save one communication

Principle of ElGamal Encryption



- ✓ This looks very similar to the DHKE! The actual Elgamal protocol re-orders the computations which helps to save one communication .

ElGamal Encryption

Example: **In this example, Bob generates the Elgamal keys and Alice encrypts the message $x = 26$.**

Alice		**Bob**
message $x = 26$		generate $p = 29$ and $\alpha = 2$ choose $k_{pr,B} = d = 12$ compute $\beta = \alpha^d \equiv 7 \bmod 29$
	$\xleftarrow{k_{pub,B}=(p,\alpha,\beta)}$	
choose $i = 5$ compute $k_E = \alpha^i \equiv 3 \bmod 29$ compute $k_M = \beta^i \equiv 16 \bmod 29$ encrypt $y = x \cdot k_M \equiv 10 \bmod 29$		
	$\xrightarrow{y,k_E}$	
		compute $k_M = k_E^d \equiv 16 \bmod 29$ decrypt $x = y \cdot k_M^{-1} \equiv 10 \cdot 20 \equiv 26 \bmod 29$

ElGamal Encryption

- Proof of Correctness:

Show that $y \cdot K_M^{-1} \bmod p \equiv x \bmod p$

$$
\begin{aligned}
y \cdot K_M^{-1} \bmod p &\equiv y \cdot (K_E^d)^{-1} \bmod p \\
&\equiv x \cdot K_M \cdot K_E^{-d} \bmod p \\
&\equiv x \cdot \beta^i \cdot (\alpha^i)^{-d} \bmod p \\
&\equiv x \cdot (\alpha^d)^i \cdot (\alpha^i)^{-d} \bmod p \\
&\equiv x \cdot \alpha^0 \bmod p \\
&\equiv x \bmod p
\end{aligned}
$$

ElGamal Encryption

- K_E must be different for every x.
- ElGamal is a probabilistic encryption scheme, unlike schoolbook RSA.
- Since it depends on DLP, p should be at least 1024 bits long.
- i, K_{pr} should result from a TRNG.

ElGamal Encryption: Attack

Choose p, α

$k_{pr} = d \in \{2,\ldots,p-2\}$

$k_{pub} = \beta \equiv \alpha^d \bmod p$

(β, p, α)

Choose $i \in \{2,\ldots,p-2\}$

$k_E \equiv \alpha^i \bmod p$

$k_M \equiv \beta^i \bmod p$

$y \equiv x \cdot K_M \bmod p$

(k_E, y)

$k_M \equiv K_E^d \bmod p$

Decrypt: $x \equiv y \cdot K_M^{-1} \bmod p$

A passive attacker's goal is to observe the channel and compute x.

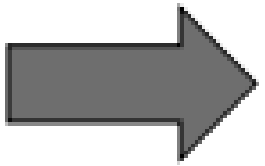
He doesn't have i or d.

ElGamal Encryption: Attack

Passive attack option 1:

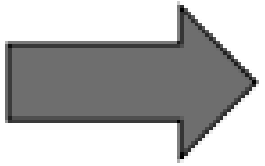
Compute $d = \log_{\alpha}\beta$

or, compute $i = \log_{\alpha}K_E$



So, we must make sure that the DLP is very hard.
i.e., length of p is ≥ 1024 bits.

Passive attack option 2:
The attacker knows that i is being reused.



Make sure there's a fresh TRN i used every time.

ElGamal Encryption: Attack

- Passive attacks
  - Attacker eavesdrops p, α, $\beta = \alpha^d$, $k_E = \alpha^i$, $y = x \cdot \beta^i$ and wants to recover x
  - Problem relies on the DLP
  - Key must be at leasst **1024 bits** long
- Active attacks
  - MITM attack defeats it
  - An attack is also possible if the secret exponent i is being used more than once
  - If attacker can guess the plaintext of one message, it can be used to decrypt another message using the same key

Example

- Find $3^{100000} \mod 53$
- Use Fermat's theorem to find a number x between 0 and 36 with X^{145} equivalent to 7 modulo 37.

Example :1

- Find $3^{100000} \mod 53$

$$\boxed{a^p \equiv a \pmod{p}}$$

$$a^{p-1} \equiv 1 \pmod{p}$$

Using Fermat Little Theorem:

$3^{53-1}=1 \mod 53$

$100000/52 : q=1923 , r=4$

$3^{99996} = (3^{52})^{1923}=1 \mod 53$

$3^{100000} = 3^4(3^{99996}) = 3^4 \mod 53$

$3^{100000} = 81 \mod 53 = 28 \mod 53$

Example :2

- Use Fermat's theorem to find a number x between 0 and 36 with X^{145} equivalent to 7 modulo 37.

Using Fermat Little Theorem:

Since 37 is prime and with x not divisible by 37,

Then $x^{37-1} = x^{36} = 1 \bmod 37$.

$x^{145} = (x^{36})^4 \cdot x = 7 \bmod 37$

Then $x = 7 \bmod 37$

Presentation Topics

- Whirlpool Hash Function
- Open-PGP CFB mode of operation
- Attacks against Open-PGP CFB mode of operation
- Homomorphic Encryption Algorithms
- Secret Sharing Protocols
- Chosen-prefix collision attack on SHA-1 Hash function

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Thank You!

See You next Lectures!!
Any Question?

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